Dependence Structure between Business Cycles and CO2 Emissions in the U.S.: Evidence from the Time-Varying Markov-Switching Copula Models

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Abstract: The relationship between CO_2 emissions and economic growth is well-examined. However, there is a gap in the literature to examine the nexus by regime-switching models. For this purpose, this paper examines the interdependence relations between CO_2 emissions and the industrial production index as a measure of business cycles at the monthly frequency in the United States. We use a new approach to modeling dependence between the underlying variables over time, combining the time-varying copula and the Markov switching models. We find that there is a significant dependence structure between business cycles and CO_2 emissions, which has a regime-switching feature, for the period from January 1973 to January 2017. Specifically, during the recession episodes, we deduce that until 1982, the high dependence regime with the Gaussian copula is valid. Since the beginning of 1983, the low dependence structure regime becomes prominent.

JEL Codes: Q44, E32, C58
Keywords: carbon dioxide emissions; business cycles; Markov-switching models; copula models; regime-dependency

1. Introduction

Due to the rising problems of global warming and climate change in the early 2000s, researchers have intensified their interest in the determinants of the related issues. The empirical studies on the subject show that climate change has mainly been related to environmental degradation (generally measured by CO_2 emissions) (Lopez-Menendez et al., 2014).[1] The recent empirical papers usually have focused on the economic growth-environmental quality (CO_2 emissions) nexus since the economic growth has been considered as a leading determinant of the environmental degradation (see, e.g., Shahbaz et al., 2017a).

At this stage, the environmental Kuznets curve (EKC) hypothesis is proposed by Grossman and Kruger (1995) to analyze the association between the level of CO_2 emissions and economic growth. According to the EKC hypothesis, CO_2 emission increase as per capita income rises, but an economy country reaches a level of the upper-middle (or high) income, the level of CO_2 emissions will fall (Gozgor and Can, 2016). However, it is essential to note that reaching the upper-middle (or high) income level does not necessarily mean that the level of CO_2 emissions in a developed economy will reduce (Atasoy, 2017; Congregado et al., 2016). At this point, policymakers must implement the necessary policy implications to decrease the level of CO_2 emissions. In other words, policymakers should find out the details of the relationship between the dynamics of economic growth and the level of CO_2 emissions.

At this point, it is noteworthy to state that testing the EKC hypothesis does not put forward the detailed policy implications to understand the effects of economic growth on CO_2 emissions. Therefore, the researchers attempt to utilize new methods to understand the CO_2 emissions-growth nexus instead of the EKC hypothesis, since the long-run forecasts of CO_2 emissions assume that economic growth is constant over time (Sheldon, 2017). In other words, the empirical papers should take the business cycle into account, i.e., one of the most

[1] Countries have recently attempted to take measures to tackle environmental degradation through the multi-country agreements (e.g. see the Paris Agreement of the United Nations).

critical elements of the macroeconomic policy (Shahiduzzaman and Layton, 2015; Sheldon, 2017). Besides, according to Bowen and Stern (2010), environmental policies should respond to the macroeconomic downturns, i.e., policymakers should take the business cycle phase into account. This issue is also not only crucial for environmental policies but also fiscal policies (i.e., government expenditures and optimal tax rates) during the times of the macroeconomic downturns (Fischer and Heutel, 2013; Fischer and Springborn, 2011). The recent idea in the empirical literature for paying regard to business cycles is related to the consequences of the global financial crisis of 2007–09 since gross domestic product (GDP) of the world economy shrank for the first time in 2009 after the Second World War (Cohen et al., 2017; Peters et al., 2012).

Our paper focuses on the U.S. economy since it is the second largest greenhouse gases emitted country in the globe in 2017 and has recently targeted a significant decline of greenhouse gas emissions by around 27% in 2025 compared to its level in 2005 (Khan et al., 2016; Shahiduzzaman and Layton, 2017). Indeed, the U.S. economy is the ideal example for analyzing the relationship between business cycles and CO_2 emissions since it is among the top countries not only for CO_2 emissions but also for per capita income. The U.S. is the top country with the highest CO_2 emissions per capita in the world for the period from 1965 to 2016, and it is the second country (China is the first) in the world in 2016 (British Petroleum (BP), 2017; Shuai et al., 2018). In addition, according to the World Development Indicators (WDI) (2018) data, the share of nominal GDP of the U.S in the world's total GDP is 24.6% in 2016[2] and the U.S. is one of the wealthiest countries with its $57,467 GDP per capita in 2016 (World Bank, 2018). In short, the U.S. economy has the significant capacity to change the pattern of the global climate change in terms of per capita income level and the level of CO_2

[2] According to the WDI data, nominal GDP of the U.S. and the World is 18.57 trillion US$ and 75.54 trillion US$ in 2016, respectively (World Bank, 2018).

emissions; and therefore, we should enhance our knowledge on the relationship between business cycles and CO_2 emissions in the U.S.

Although our paper analyzes the relationship between CO_2 emissions and business cycle in the U.S. economy, various papers have focused on the EKC hypothesis for examining the relationship between per capita income (GDP) and CO_2 emissions in the U.S (see, e.g., Congregado et al., 2016). However, there is still no paper that analyzes the related relationship in a business-cycle framework by using the Markov-Switching time-varying copula models.

To this end, our paper aims to analyze the dependence structure between the level of CO_2 emissions and economic performance in a business cycle framework in the U.S. for the period from January 1973 to January 2017 by using the Markov-Switching time-varying copula models. It is important to note that the Markov-Switching and the time-varying copula models take nonlinearity and asymmetry into account. They are also able to capture the different asymmetry characteristics of the time-series in the forms of the high-dependence, the normal-dependence, and the low-dependence. At this stage, it is interesting to consider tail dependence in the growth-CO_2 emissions nexus since there could be various policy implications following the tail dependence. At this point, our paper aims to fill the related gaps in the empirical literature by analyzing the case of the U.S. Our empirical analysis captures the data spanning the period from January 1973 to January 2017, which covers the oil price shocks in the 1970s and the global financial crisis of 2007–09.

To the best of our knowledge, our paper is the first to analyze the relationship between business cycles and CO_2 emissions by utilizing the Markov-Switching the time-varying copula models; and actually, this is the novel contribution of our paper to the existing empirical literature. Specifically, we aim to analyze how macroeconomic shocks, which determine the business cycle fluctuations in industrial output, affect the pro-cyclicality of the carbon dioxide emissions. Overall, our paper uses new and robust methods to analyze the relationship between business cycles and CO_2 emissions. The empirical results show the importance of business cycles on the dependence structure. The empirical findings also illustrate that CO_2 emissions in the U.S. are regime-dependent.

The remainder of the paper is organized as follows. Section 2 briefly reviews the previous literature on the relationship between CO_2 emissions and economic performance in a business cycle framework. Section 3 explains the methodological issues of the estimation procedure as well as describing the estimation of the model and the data. Section 4 provides the empirical results and also discusses the policy implications. Section 5 concludes.

2. Literature Review

Previous studies that examine the relationship between the level of CO_2 emissions and per capita income (or economic growth) both in the developing economies and the developed countries are mostly based on the EKC hypothesis (see e.g., Congregado et al., 2016; Pao and Tsai, 2010; Shahbaz et al., 2017b). At this stage, there are only a few papers that consider the role of the business cycle in CO_2 emissions-growth nexus using the time-series data.[3] For example, using the monthly frequency data for the period from 1973 to 2000 in the U.S., Thoma (2004) illustrates that there is the asymmetric response of electricity consumption to change of industrial production over a business cycle phases. Considering both the monthly and the quarterly frequency data for the period from 1981 to 2003, Heutel (2012) demonstrates that the level of CO_2 emissions is pro-cyclical in the U.S., i.e., it increases during the expansion times and decreases during the contraction times. Also, Peters et al. (2012) use the annual data for the world economy, and they also split the data as the

[3] There are also panel data studies to analyze the relationship between CO_2 emissions and business cycles (see Burke et al., 2015; Doda, 2014; York, 2012). In addition, refer to Cohen et al. (2017 and 2018) and Zhao et al. (2016) for possible explanations of cross-country differences in the relationship between CO_2 emissions and business cycles.

developing- and the developed economies for the period from 1960 to 2010. The evidence that the suppressing effect of the global financial crisis of 2007–09 on the level of CO_2 emissions is temporary and CO_2 emissions rebounded in 2010. Using the decomposition analysis, Jotzo et al. (2012) also focus on the annual data for the world economy for the period from 1972 to 2010 and observe that the rising energy intensity caused to the rebound of CO_2 emissions in 2010. Using the decoupling analysis, Wang et al. (2018) compare the decoupling strategies of economic growth from carbon dioxide emissions in China, and the U.S. Rodríguez et al. (2018) also provide different decoupling environmental strategies in 28 European Union (EU) countries using the data from 1950 to 2012.

Furthermore, Shahiduzzaman and Layton (2015) focus on the U.S. business cycles for the period from 1949 to 2011 with the annual data. They also use the monthly data for the period from 1973 to 2013. Using the decomposition analysis with the time-series techniques, they find that total CO_2 emissions fall much faster during the contraction periods than they increase during the expansion periods. In their empirical paper, Shahiduzzaman and Layton (2017) enhance the related evidence (i.e., the reduction in CO_2 emissions is higher in the contraction periods than the expansion periods) by using the data for the period from 1973 to 2014. They also discuss the policy implications in the U.S. to achieve the 2025 target for reducing the greenhouse gas emissions; and thus, the slowing down the pattern of climate change in the world. The similar evidence for CO_2 emissions-business cycles nexus is obtained by Eng and Wong (2017) using the nonlinear autoregressive-distributed lag (NARDL) model estimations in the USA. Finally, using the quarterly frequency data for the period from 1960 to 2011, Sheldon (2017) demonstrates that there is the asymmetric response of CO_2 emissions to business cycles in the U.S. In other words, the level of CO_2 emissions reduces much more when per capita income falls than they rise when per capita income increases.

To conclude the literature review, we observe that there are only a few studies for analyzing how business cycles affect CO_2 emissions. Most of the studies have focused on the U.S. economy due to its pivotal role in terms of the world's GDP and CO_2 emissions. Following them, our paper also examines the case of the U.S. economy within the monthly dataset for the period from January 1973 to January 2017; thereby, aims to determine the exact specification of business cycle turning points in the U.S. economy. The monthly and the updated dataset shed additional lights to our knowledge of the association between business cycles and CO_2 emissions in the U.S. Besides, most of the papers have used the traditional time-series techniques to analyze the relationship the related variables. Different from other studies in the literature, our paper is the first to analyze the dynamic dependence between business cycles (fluctuations in aggregate economic activity) and climate change (measured by CO_2 emissions). For this purpose, we utilize the regime switching copula-based autoregressive (AR)–exponential generalized autoregressive conditional heteroskedasticity (EGARCH) approach, which takes into account the nonlinearity and the asymmetric characteristics of the time-series. In the empirical analysis, we also consider the Markov-switching dynamic (autoregressive) copula approaches since they are flexible to capture the asymmetry in the forms of the high-dependence, the normal-dependence, and the low-dependence. Indeed, the findings indicate the importance of business cycles on the dependence structure. Furthermore, the level of CO_2 emissions in the U.S. is regime-dependent.

3. Methodological Issues

The analysis is performed in two steps: (i) the marginal distribution for each random variable is obtained through the AR-EGARCH models with a skewed t-distribution and (ii) the regime switching copula models are used to model the dependence. To measure the dynamic dependence between fluctuations in the measures of economic activity and climate change, we use the regime switching copula based on the AR-EGARCH approach, which takes into account the asymmetry and the nonlinearity. The Markov-switching dynamic (autoregressive) copula approach is flexible to capture the asymmetry in the form of high dependence and the low (or normal) dependence. Firstly, the AR(p)-EGARCH(1,1) model is adopted to identify the marginal distributions of each data series, $F_i, i = 1,2$, which is used as input data for the regime-switching copula model. Secondly, the dependence between the two marginal distributions, which deduce from the AR(p)-EGARCH(1,1) model, can be estimated by a regime switching copula model. We explain the related procedure as follows.

3.1. AR-EGARCH Model

The marginal distributions of CO_2 emissions and industrial production are characterized by an AR (p)-EGARCH (1,1) with the student's t-distribution innovation model that accounts for the time-varying volatility. The EGARCH model also has added the benefit to take into account the asymmetric effect of positive and negative shocks, so that the EGARCH model allows for testing of asymmetries. The AR (p)-EGARCH (1,1) with the skewed student's t-distribution innovation can then be written as follows:

$$r_t = \phi_C + \sum_{j=1}^{p} \phi_j r_{t-j} + \varepsilon_t \quad (1)$$

where ϕ_j is the autoregressive parameter of r_t, ε_t is an error term, p is a non-negative integer. The time-varying volatility can be specified by the t-EGARCH (1, 1) process, which can be characterized by the following equations (Nelson, 1991):

$$\varepsilon_t = \sigma_t z_t \quad (2)$$

$$ln(\sigma_t^2) = \omega + \beta ln(\sigma_{t-1}^2) + \gamma \left[\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \right] + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \tag{3}$$

where $\varepsilon_{it}, i = 1, \dots, T$ are i.i.d random variables, σ_t^2 is the conditional variance given past information, ω, β, α> 0 and $\alpha + \beta < 1$ assuring $\sigma_t^2 > 0$. z_t is the standardized residual. α parameter illustrates the "GARCH" effect (the symmetric effect of the model). β measures the persistence in conditional volatility (the lagged conditional variance) (Engle, 1982). γ parameter captures the asymmetric or the leverage effect. If γ = 0, then the model is symmetric. When γ < 0, then positive shocks produce less volatility than negative shocks. When γ > 0, it implies that positive innovations are more fluctuating than negative ones. We also assume that ε_{it} follows a skewed-t distribution (with v and γ degrees of freedom and asymmetry, respectively), which has the following density:

$$g(z|\nu, \gamma) = \begin{cases} bc\left(1 + \frac{1}{\nu-2}\left(\frac{bz+a}{1-\gamma}\right)^2\right)^{-(\nu+1)/2} & z < -a/b \\ bc\left(1 + \frac{1}{\nu-2}\left(\frac{bz+a}{1+\gamma}\right)^2\right)^{-(\nu+1)/2} & z \geq -a/b \end{cases} \tag{4a}$$

where the constants *a*, *b* and *c* are obtained from:

$$a = 4\lambda c\left(\frac{\nu-2}{\nu-1}\right), \qquad b^2 = 1 + 3\lambda^2 - a^2, \quad c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)} \tag{4b}$$

3.2. Copula Models

3.2.1. Basic Concepts

An n-dimensional copula $C(u_1, u_2, \dots, u_n)$ is a multivariate distribution function in $[0,1]^n$; and therefore, each marginal distribution is uniform on the interval $[0,1]$ (Schweizer and

Sklar, 1983). Given *n* random variables $x_1, x_2, \ldots, x_n$ with joint distribution, $H(x_1, x_2, \ldots, x_n)$ moreover, with marginal functions $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$:

$$H(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)\,) \qquad (5)$$

There exists a unique copula function C implied by the continuity of $F_1, F_2, \ldots, F_n$:

$$C(u_1, u_2, \ldots, u_n) = H\left(F_1^{(-1)}(u_1), \ldots, F_n^{(-1)}(u_n)\right) \qquad (6)$$

Where$u_1 = F_1(x_1), \ldots, u_n = F_n(x_n)$.

$F_1, F_2, \ldots, F_n$ and C are n-differentiable, then the joint density function can be obtained as follows:

$$h(x_1, x_2, \ldots, x_n) = c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)\,)\prod_{i=1}^{n} f_i(x_i) \qquad (7)$$

where h is the density function of the joint distribution H, f_i is the marginal density function and c is copula density, which is obtained by differentiating equation-5:

$$c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)\,) = \frac{h\left(F_1^{(-1)}(u_1), \ldots, F_n^{(-1)}(u_n)\right)}{\prod_{i=1}^{n} f_i(F_i^{(-1)}(u_i))} \qquad (8)$$

Copula functions often used in finance literature are associated with a quadratic form of correlation between the marginal (elliptical forms) as the Gaussian and the Student's t copula models. The distribution functions of this copula family allow for asymmetric left- and right-tail dependence. Other copula approaches that have been used, allowing for asymmetric dependence as Clayton and the Gumbel type copula models, which allow for the upper tail and lower dependence respectively. Hence, there is a variety of copula functions with specific dependence structure, making a comparison between the different functional forms of copula impossible. For that reason, we turn to the tail dependence as a dependence measure to make this comparison possible. We define the copula models as follows:

Definition (Nelsen, 2006): Let X_1 and X_2 be continuous random variables with distribution functions F and G, respectively. *The upper-tail dependence parameter*λ_U is the limit (if it exists) of the conditional probability that X_2 is greater than 100t$^{\text{-th}}$ percentile of G given that X_1 is greater than the 100t$^{\text{-th}}$ percentile of F as t approaches 1, i.e.

$$\lambda_U = \mathrm{Lim}_{t\to 1^-} P\left(X_2 > G^{(-1)}(t) \middle| X_1 > F^{(-1)}(t)\right) \qquad (9)$$

Similarly, *the lower tail dependence parameter*λ_L is the limit (if it exists) of the conditional probability that X_2 is less than or equal to the 100t$^{\text{-th}}$ percentile of G given that X_1 is less than or equal to the 100t$^{\text{-th}}$ percentile of F as t approaches 0, i.e.

$$\lambda_L = \mathrm{Lim}_{t\to 0^+} P\left(X_2 \leq G^{(-1)}(t) \middle| X_1 \leq F^{(-1)}(t)\right) \qquad (10)$$

These parameters are nonparametric and depend only on the copula of X_1 and X_2, as the following theorem demonstrates.[4]

Theorem: Let X_1, X_2, F, G,λ_U and λ_L be as in precedent Definition, and let *C* be the copula of X_1 and X_2, with the diagonal section δ_C. If λ_U and λ_L exist, then

$$\lambda_U = 2 - \underset{t\to 1^-}{\mathrm{Lim}} \frac{1-C(t;t)}{1-t} = 2 - \delta'_C(1^-) \qquad (11a)$$

[4] For proof, see Nelsen (2006).

$$\lambda_L = \lim_{t \to 0^+} \frac{C(t;t)}{t} = \delta_C'(0^+) \quad (11b)$$

If λ_U is in (0,1], we say C has the upper tail dependence; if λ_U = 0, we say C has no upper tail dependencies; and similarly forλ_L.

Hence, the tail dependence is entirely defined by the related copula and is not affected by the marginal distribution variation. The use of tail dependence measures makes it possible to examine, which model is capable of reproducing stylized facts about the relationship between CO_2 emissions and industrial production. Moreover, the tail dependence can be defined as the probability that a pro-cyclicality or counter-cyclicality happens, given CO_2 emissions during economic booms and recessions. Different functional forms for copula that can be used (Nelsen, 2006). Our study test five copula functions, and their details are provided in Table-1.[5]

Table-1. Details of the Time-varying Markov Switching Copula Models

Copula Model	Type	Parameters	Tail Dependence
Gumbel (G)	Archimedean	Correlation θ	Only Upper
Rotated Gumbel (RG)	Archimedean	Correlation θ	Lower
Symmetrized Joe-Clayton (SJC)	Archimedean	The Dependence Parameters τ^U and τ^L are the Measures of Dependence of the Upper- and Lower Tail, respectively.	Both Upper and Lower
Normal (N)	Elliptical	The Linear Correlation ρ	None
Student's t (S)	Elliptical	The Linear Correlation ρ	Symmetric

Note: For the relevant technical details of the procedures to obtain Markov switching copula parameter estimates, refer to Nelsen (2006).

3.2.2. Regime-Switching Copula Models

We observe the evidence that dependency between CO_2 emissions and industrial production does not stay constant over time. Hence, the dependence between CO_2 emissions, r_{1t}, and industrial production, r_{2t}, is allowed to vary over time. Following Patton (2006), we let the

[5]The details of the copula functions employed in our study are defined in Table-1.

dependency parameter evolve according to an AR (p) process, for the copula function dependency parameters. We assume that the functional form of copula remains constant, but copula parameters can evolve over the period under concern. Considering $R_t = (R_{1t}, R_{2t})$ $t = 1,2, \ldots$ the copula-based EGARCH model can be represented as follows:

$$H(R_t|\mu, \boldsymbol{h_t}) = C_{\phi_C}\big(F_1(R_{1t}|\mu, h_{1t}), F_2(R_{2t}|\mu, h_{2t})\big) \tag{12}$$

where $F_i(R_{it}|\mu, h_{it}), i = 1,2$ the marginal distributions are specified as standard univariate EGARCH processes and C_{ϕ_C} is the copula function with time-varying dependence parameter ϕ_{Ct}, which switching according to a first-order Markov Chain:

$$\phi_{Ct,S_t} = \Lambda\big(\omega_C^{S_t} + \beta_{Ct-1}\phi_{Ct-1} + \psi_t\big) \tag{13}$$

where Λ is a logistic transformation of each copula isa function to constrain the dependence parameter in a fixed interval[6], S_t are the Markov states which assumed to be two states of the economy, namely expansion and recession regimes. Then, the transition probability π is defined as a 2x2 matrix of p_{ij} $i,j = 1,2$ where p_{ij} is the probability of moving from state i to state j:

$$\pi = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} = \begin{pmatrix} p & 1 - p \\ 1 - q & q \end{pmatrix} \tag{14}$$

ψ_t represents a "forcing variable," defined as the mean absolute difference between $u_1 = F_1(x_1)$ and $u_2 = F_2(x_2)$ for the Gumbel (G), the Rotated Gumbel (RG), and the Symmetrized

[6] For example, the logistic transformation keeps the upper- and lower tails of the SJC copula bounded to (0,1).

Joe–Clayton (SJC) copula models, and the mean of product between u_1 and u_2 for the Normal (N) and Student's t (S) (Patton, 2006).[7]

3.3. Estimation Procedure

The maximization of the log-likelihood function can estimate the copula and the marginal density parameters:

$$l(\boldsymbol{\theta}|\boldsymbol{R_t}) = \sum_{t=1}^{T} log\left(\left(C_{\phi_C}\left(F_1(R_{1t}|\mu, h_{1t}), F_2(R_{2t}|\mu, h_{2t})\right)\middle|\phi_C, S_t\right) \times \prod_{i=1}^{2} f_{it}(R_{it}|\theta_i)\right) \quad (15)$$

Where $\theta_i = \mu_i, h_{it}, i = 1,2$ and $\boldsymbol{\theta}$ is a vector with all model parameters. The log-likelihood is a separable function; then we can use the two-step maximum likelihood estimation the procedure called the inference function for margins (IFM) technique proposed by Joe and Xu (1996). This two-step procedure consists of estimating the parameters of the univariate marginal distributions modeled as the univariate EGARCH process in a *first step* and then using these estimates to estimate copula parameters by maximizing the log-likelihood function in a *second step*. However, the dependence parameter estimation via copula depends on a non-observable state (S_t), which follows a Markov chain. Accordingly, the estimation of the regime-switching copula parameters requires inferences on the probabilistic evolution of the state variable (S_t), and that's why we build our estimation approach on the smoothing algorithm of Kim (1994).

3.4. Data Description and Preliminary Analysis

In our study, climate change is measured by the level of carbon dioxide emissions, and the data are obtained from the U.S. Energy Information Administration. The output level has been

[7] For the RG and the SJC copula models, the forcing variable is the mean absolute difference between u_1 and u_2 given by $\alpha_C.\frac{1}{p}\sum_{j=1}^{p}|u_{1,t-j} - u_{2,t-j}|$, while it is defined as the mean of the products between $\alpha_C.\frac{1}{p}\sum_{j=1}^{p}\Phi^{-1}(u_{1,t-j}).\Phi^{-1}(u_{2,t-j})$ and $\alpha_C.\frac{1}{p}\sum_{j=1}^{p}T_v^{-1}(u_{1,t-j}).T_v^{-1}(u_{2,t-j})$ for the Normal and the Student's t copula models across ten previous periods (Patton, 2006).

measured by the index of industrial production, which was obtained from the Organization for Economic Co-operation and Development (OECD) statistics online database. The empirical analysis captures the data spanning the period from January 1973 to January 2017. In Figure-1a and Figure-1b, we provide the plots of the related time-series.

[Insert Figures-1a and 1b around here]

A summary of statistics for the logarithmic returns is reported in Table-2, which shows that the growth rate of CO_2 emissions is negative, while the growth rate of industrial production index is positive. Furthermore, the evidence indicates that CO_2 emissions are relatively more volatile, and both series are negatively skewed, as well as they have high kurtosis. The null hypothesis of the normality has been rejected for both variables, and both series are stationary as evident from the unit root and stationarity tests.

Table-2. Descriptive Statistics Properties of Monthly CO_2 Emissions and Industrial Production Index (INPRO) Returns (January 1973– January 2017)

Test Statistics	CO_2Emissions	INPRO
Mean	–0.000540	0.001565
Median	0.005064	0.001901
Maximum	0.134595	0.020311
Minimum	–0.200210	–0.043900
Standard Deviation	0.052106	0.007200
Skewness	–0.562184	–1.340810
Kurtosis	3.738584	9.133563
Q(10)	422.03***	209.20***
$Q^2(10)$	31.122***	141.55***
Jarque-Bera Statistics	39.81361***	985.8571***
PP Test	–50.9717***	–17.8768***
ADF Test	–5.9352***	–8.0512***
KPSS Test	0.0533	0.1096

Notes: The Q(10) and the Q^2 (10) refer to the Ljung-Box tests for autocorrelation, respectively. The ADF, the PP, and the KPSS are the empirical statistics of the Augmented Dickey-Fuller (1979), and the Phillips-Perron (1988) unit root tests, and the Kwiatkowski et al. (1992) stationarity test, respectively. *** denotes the rejection of the null hypotheses of normality, no autocorrelation, unit root, non-stationarity, and conditional homoscedasticity at the 1% significance level

4. Empirical Findings

4.1. Results of the Marginal Model

Table-3 provides the estimated effects of the marginal models. We select the optimal model, AR (p)-EGARCH (1,1), according to the Schwarz / Bayesian Information Criterion (BIC) criteria. The estimation shows that the asymmetry coefficient is negative and statistically significant for the industrial production index (IPI) series, and it is positive as well as statistically significant for CO_2 emissions. This evidence argues a heavy tail to the left for the marginal distribution of IPI, which highlights that negative shocks produce less volatility than positive ones. The heavy right tail of CO_2 emissions marginal distribution implies that the positive innovations are more fluctuating than the negative ones. Therefore, we reject the normal distribution as an adequate fit for our series.

Table-3. Results of the Estimations of the Marginal Distribution Models

Mean	CO_2 Emissions	INPRO
$\emptyset_0$	4.775***	0.172***
	(1.467)	(0.057)
$\emptyset_1$	–0.437***	0.216***
	(0.047)	(0.009)
$\emptyset_2$	–0.118**	0.202***
	(0.051)	(0.027)
$\emptyset_3$	–0.128***	–
	(0.044)	
$\emptyset_4$	–0.173***	–
	(0.045)	
$\emptyset_5$	0.031	–
	(0.036)	
$\emptyset_6$	–0.277***	–
	(0.037)	
Variance	CO_2 Emissions	INPRO
$\boldsymbol{\omega}$	0.417**	–0.612***
	(0.174)	(0.125)
$\boldsymbol{\alpha}$	0.307***	0.389***
	(0.079)	(0.049)
$\boldsymbol{\beta}$	0.763***	0.704***
	(0.064)	(0.087)
Tail	19.793	6.536***
	(16.204)	(1.749)
Log Likelihood	–1463.39	–455.1744
Akaike Information Criteria	5.657	1.765
Bayesian Information Criterion	5.763	1.838
Q^2 (10)	17.47	11.38
	(0.047)	(0.329)

Q^2 (20)	14.52	12.69
	(0.123)	(0.304)
ARCH LM (10)	9.97092	5.92096
	(0.44305)	(0.82186)
Kolmogorov–Smirnov	0.999	0.999
Anderson–Darling	0.0007954	4.319e-13
Cramér–von Mises	0.003548	1.524e-08
Berkowitz	1.320312e-07	1.320312e-07

Notes: The Table presents the estimates of univariate the AR-EGARCH model. The standard errors are in parentheses. Q^2 denotes the Ljung-Box p-values for serial in the squared residual model calculated with 10 and 20 lags. The last four rows show goodness-of-fit tests to the probability integral transform from the margins (Kolmogorov–Smirnov, Anderson–Darling (Anderson and Darling, 1952), Cramér–von Mises, and Berkowitz (Berkowitz, 2001) tests. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Also, the estimated coefficients (β) exhibit high values indicate that volatility was persistent across our underlying macroeconomic variables. Then, the use of the multivariate normal or the symmetric Student's t distributions would not be an appropriate choice to characterize the joint distribution of industrial production and CO_2 emissions returns. We also observe that the results of the goodness-of-fit tests, which provide no evidence of serial correlation, as we can infer by the Ljung-Box Q^2 statistics and the ARCH effects. Furthermore, the Anderson–Darling (AD), Berkowitz, Cramér–von Mises, and Kolmogorov–Smirnov (KS) tests, which are used as uniformity tests for the transformed marginal of these residuals provides the mixed results. Those are the formal statistics to ensure the adequacy of the potential switching behavior; therefore, we should use the copula models.[8]

4.2. Results of Time-varying (Dynamic) Copula Dependence

After the estimations of the marginal distribution, the four time-varying Copula functions (time-varying Normal, time-varying Student's t, time-varying Rotated Gumbel, and time-varying Gumbel)[9] are used to broadly analyze the dynamic dependence structures between business cycles and CO_2 emissions. Moreover, the Markov-Switching Copula models are

[8] Note that there is no potential confusion between switching behavior and structural breaks in the dependence relationship over the long sample period under concern.

[9] Note that the time-varying Symmetrized Joe-Clayton (SJC) Copula estimations do not converge.

employed to examine further if the dependence structures of CO_2 emissions will be regime-switching (the high- and low regimes). It is common in the literature to select the best model as the one presenting the highest likelihood figure and the lowest information criteria, such as the Akaike Information Criteria (AIC) and the BIC values. The ability of time-varying Copula function to predict in-sample the dependence between industrial production and CO_2 emissions is judged by the log-likelihood (LL) value of the competing models. The time-varying Normal Markov copula emerges as the best fitting models (have the highest likelihood figure), which also has the lowest AIC and BIC values. This finding suggests that the dependence structure between the time series variables is seemingly symmetric, i.e., there is no noteworthy difference between the degrees of tail dependence in the upper- and lower tails.

The estimations of the Markov Switching Copula models further reveal that the dependence structure between the two variables is not unchanging, but switches through different regimes. In Table-4, we display the copula estimation results for the time-varying Normal Markov copula function (with the regime switching) used in our study. We also report other time-varying Markov copula functions (the Gumbel, the Rotated Gumbel, and the Student's t) in Appendix Table-I and Appendix Figure-I. However, it is essential to note that the results of the time-varying normal Markov copula are considered as the baseline results because Log Likelihood criteria is a maximum for the time-varying Normal Markov copula function.

Table-4. Time-varying Normal Markov Copula Estimation Results for Industrial Production and CO_2 Emissions

Parameters	Dependent Variable: CO_2 Emissions
ω^0	2.1557*** (0.3626)
ω^1	

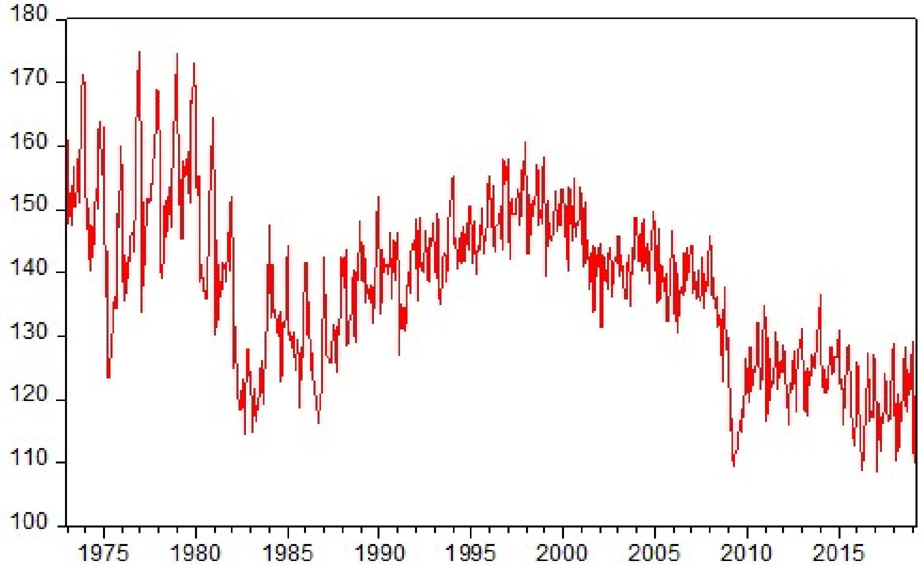
	-1.9327***
	(0.3488)
β	-2.4448***
	(0.1384)
α	-0.3561***
	(0.3184)
P	0.0297
	(0.9012)
Q	0.9703***
	(0.7095)
Log Likelihood	-15.872

Notes: The numbers in parentheses represent the standard errors. *** indicates the statistical significance at the 1% level.

In the results in Table-4, the first regime characterized by low dependence (recession) is less persistent than the second one described by high dependence (expansion), as indicated by probabilities p (low value) and q (high value). This result implies that there is a strong persistence in the dependence structure between CO_2 emissions and industrial production in the expansion regime. Thus, we can infer *regime 0* to be the low dependence regime, while *regime 1* is the high dependence regime. It looks that the dependence structure between CO_2 emissions and business cycles differs under the two regimes. Moreover, the dependence between the two variables has high-level fluctuations, and CO_2 emissions are more volatile than industrial production. This finding is somehow consistent with the findings of Doda (2014) and Heutel (2012).

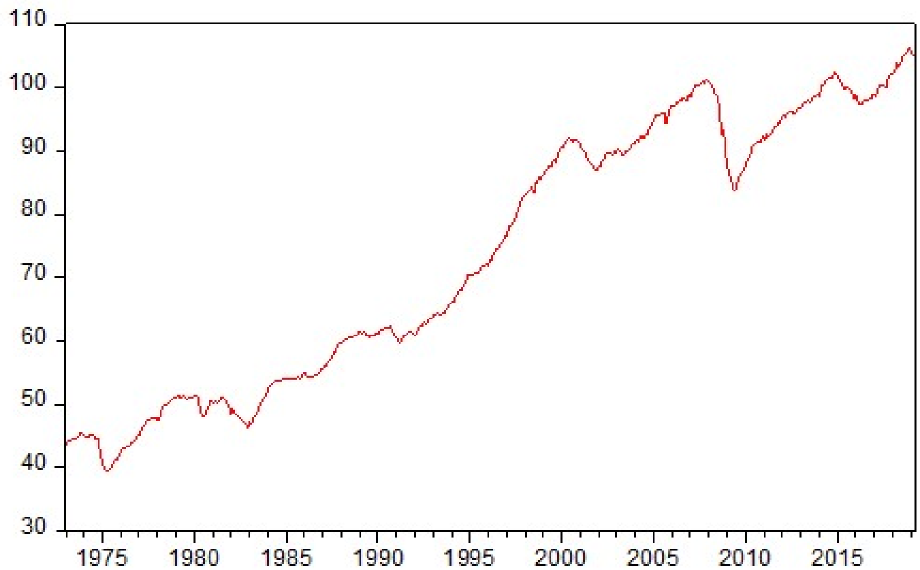
In Figure-2a, we report the time-varying dependence between the related variables. The findings show that most often, the time-varying dependence structure between industrial production and CO_2 emissions is more likely to be in the regime 1; however, during specific periods, the tail dependence between the two related variables will no longer prevail, which corroborate that the dependence structure can be characterized well by the time-varying normal copula (see, Figure-2b). The dependence structure could change over time or alternate between symmetric form without the tail dependence or with the low dependence (Da Silva Filho et al. 2012).

Figure-1a. Total Industrial Sector CO_2 Emissions (January 1973–March 2019)



Data Source: U.S. Energy Information Administration

Figure-1b. Industrial Production Index (January 1973– March 2019)



Data Source: OECD

Figure-2a. Time-varying Dependence between Industrial Production and CO_2 Emissions (Time-varying Normal Markov Copula Estimation)

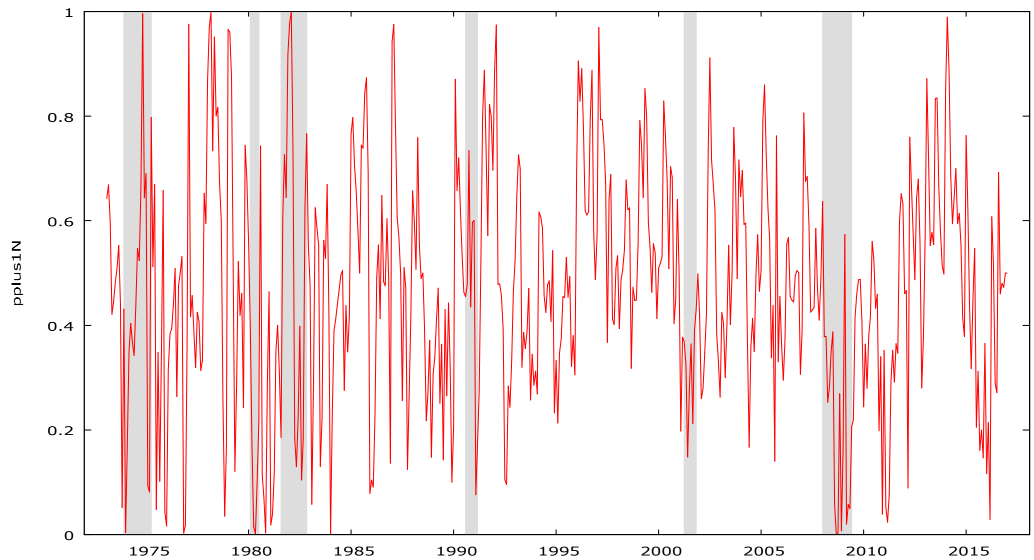
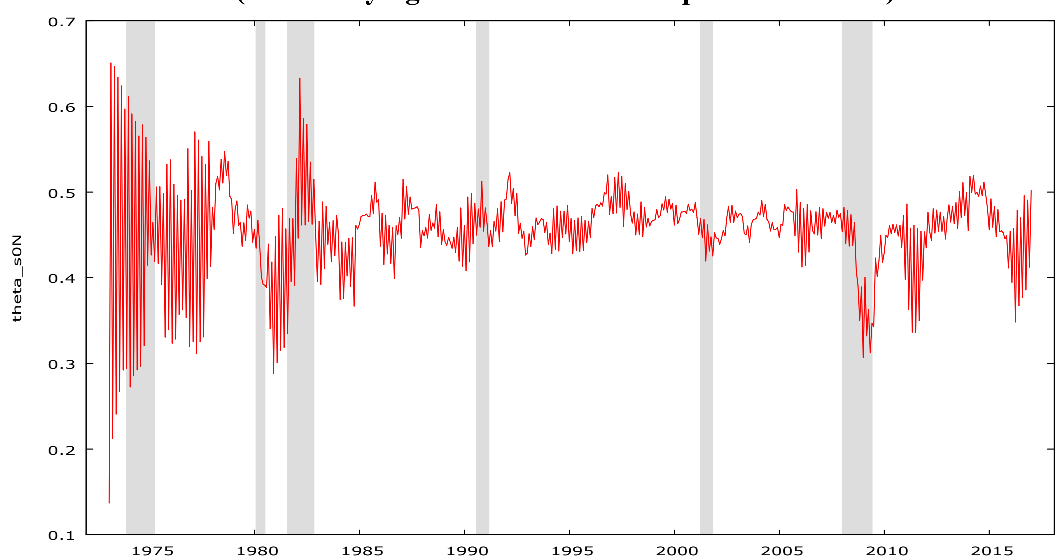


Figure-2b. Tail Dependence between Industrial Production and CO_2 Emissions (Time-varying Normal Markov Copula Estimation)



Essentially, our analysis suggests that the dependence structures between business cycles and carbon emissions tend to be different during two interesting periods. First, before 1983, we observe that the high dependence regimes characterize the recession periods; however, after this date, the dependence relation in both pairs under analysis "jumps" to a lower dependence regime during the recession phases (see Figure-2a). This result is confirmed by the significant change in the intercept term in the equation describing the dependence dynamics (see Table-3), which highlight that the dependence is the non-linear and the time-varying. Doda (2014) states that in the U.S., among other countries, the growth rate of carbon dioxide emissions is more strongly associated with the GDP growth rate during recessionary periods than expansionary periods, which is not in line with our finding as for the recession episodes the low dependence regime characterized after 1983.

Concerning the dynamic dependence captured by the estimated copula (see Figure-2b), it can be seen that starting from 1983, the dependence degree changes from values around 0.15 and 0.65 with the values around 0.3 and 0.5. Therefore, since this period there has been a decrease in the dependence, probably associated with the steady decline in oil consumption from 1979 through 1983 resulting, at least in part, from a higher petroleum price.

4.3. Discussion and Policy Implications

In this paper, we determined that 1983 is the turning point in the U.S. economy for the association between business cycles and CO_2 emissions. This evidence is the first evidence for the dynamic dependence between business cycles (fluctuations in aggregate economic activity) and climate change (measured by CO_2 emissions) using the Markov regime switching copula-based AR-EGARCH approach, which takes into account nonlinear and the asymmetric characteristics of the related time-series. We observed that business cycles, which are important for the dependence structure and CO_2 emissions in the U.S., are regime-dependent.

We also found that CO_2 emissions are pro-cyclical in the U.S. economy, i.e., CO_2 emissions increase during the expansion times and decrease during the contraction times. However, the decline in CO_2 emissions during the contraction periods is higher than the increase in CO_2 emissions during the expansion periods. Thus the relationship is asymmetric. These findings are in line with the results of Heutel (2012), Shahiduzzaman and Layton (2015 and 2017), and Sheldon (2017). Also, according to our results, business cycles are related to the consequences of the global financial crisis of 2007–09 do not significantly affect the level of CO_2 emissions. This finding is in line with the evidence of Peters et al. (2012).

Following these results, policymakers can implement some policy implications to decrease in CO_2 emissions. First, we observed that the relationship between business cycles and CO_2 emissions is asymmetric (dynamic) and regime-dependent (nonlinear). Therefore, policymakers should not use the EKC hypothesis for the long-run forecasts of CO_2 emissions in the U.S. since the EKC hypothesis assumes that economic performance is constant over time. In other words, the EKC hypothesis can be suitable only for the short-run projections of CO_2 emissions in the U.S. At this stage, business cycles, most essential elements of the macroeconomic policy and environmental policies, should respond to the macroeconomic downturns; and therefore, the U.S. policymakers should take the business cycle phase into account when they implement environmental policies. For instance, achieving the U.S. 2025 target for reducing greenhouse gas emissions can slow down the pattern of global climate change.[10] At this stage, environmental policies (e.g., providing the energy efficiency and energy intensity) and carbon taxes can help to achieve the U.S. 2025 target for reducing the greenhouse gas emissions. Renewable energy can also help to decrease carbon dioxide emissions in the U.S. (Gozgor, 2018). In addition, providing the optimal government expenditure level and tax rate during the times of macroeconomic downturns can also indirectly affect CO_2 emissions via business cycles.

[10] In here, the role of technological advances in the industry can be estimated to see whether the U.S. will achieve 2025 target using the data for the decrease in CO_2 emissions since 2007.

5. Conclusion

In this paper, we investigated the time-varying dependence and the tail dependence relationships between business cycles (measured by industrial production) and climate change (measured by CO_2 emissions) in the U.S. for the period from January 1973 to January 2017. To this end, we consider five approaches to model interdependence between the underlying variables over time, combining time-varying copula and the Markov-switching models: The Gumbel, the Rotated Gumbel, the Normal, the Student's t, and the Symmetrized Joe-Clayton. The empirical findings from the model estimations indicate that there are significant time-varying dependence and the tail dependence structures between business cycles and CO_2 emissions for the period from January 1973 to January 2017. We observe that the time-varying normal Markov copula estimation is the best-fitting model. Specifically, during the recession episodes, we find that the high dependence regime with the Normal (Gaussian) copula is valid until 1982. In addition, the beginning of 1983, the low dependence structure regime becomes prominent.

Overall, our paper demonstrates that there is a significant dependence structure between business cycles and CO_2 emissions, and it is regime-switching for the period from January 1973 to January 2017. Future papers on the subject can focus on other large economies in the world (e.g., Brazil, China, India, and Japan) to examine the interdependence relationship between CO_2 emissions and business cycles (including the role of technological advances in industries) using the time-varying copula and the Markov switching models with the stochastic volatility models.

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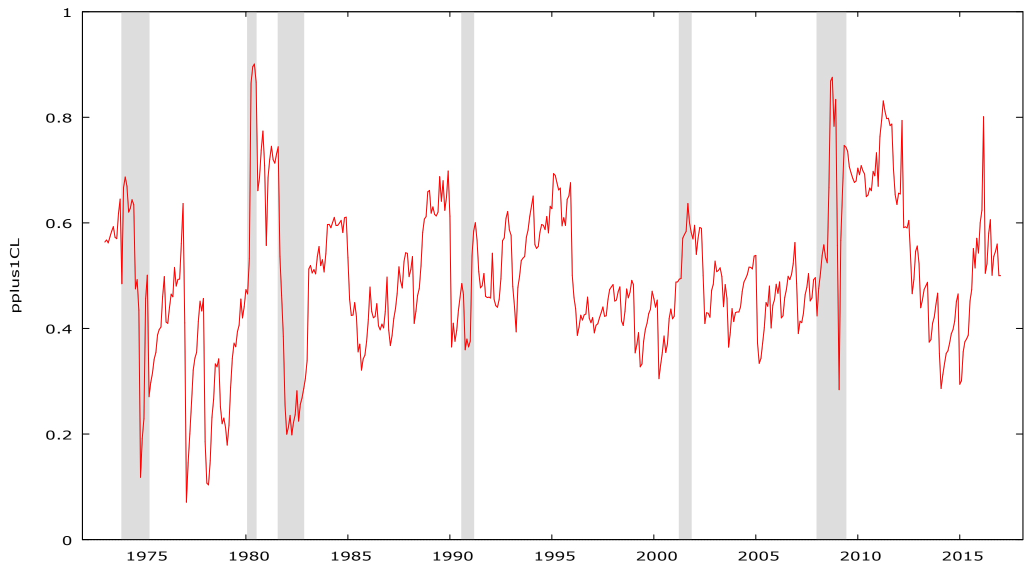
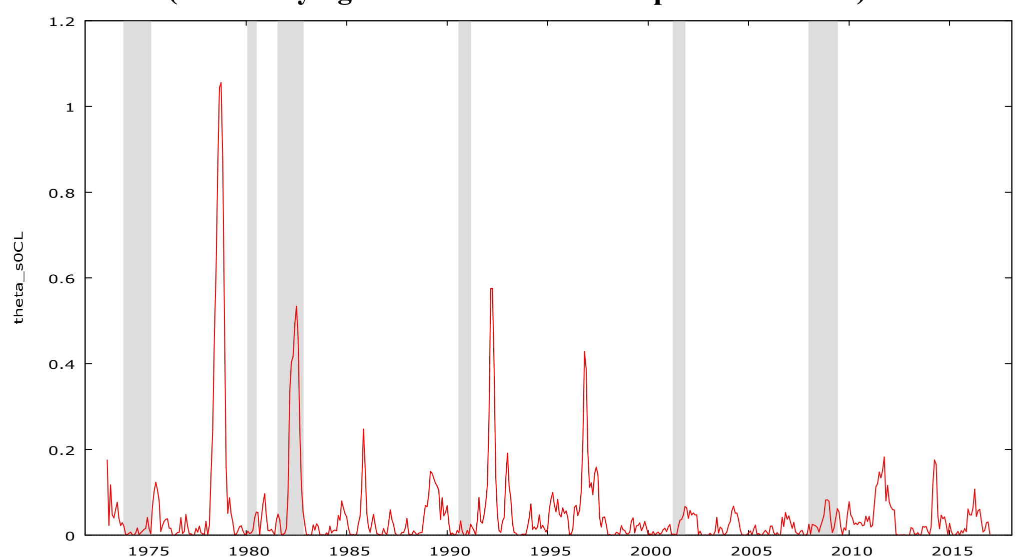
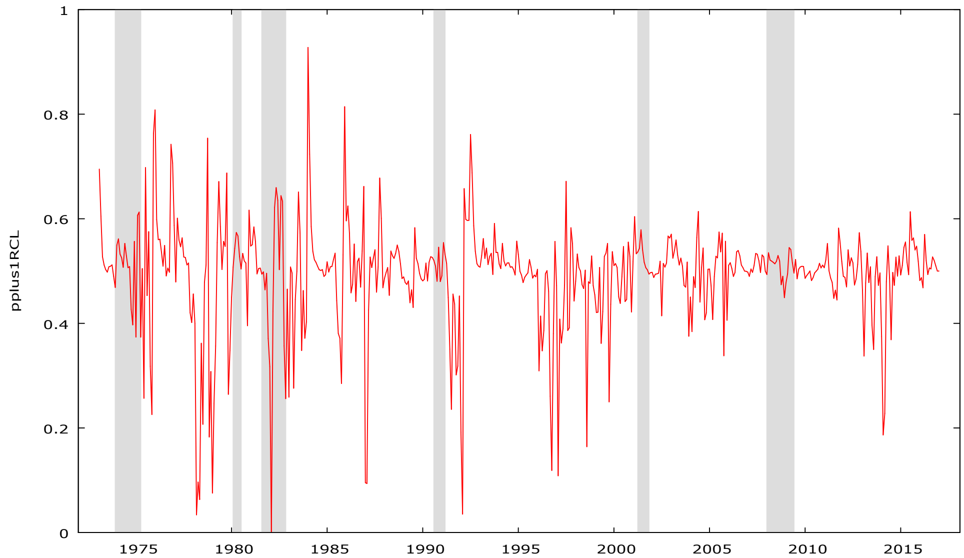
Appendix Figure-I.
Time-varying Dependence between Industrial Production and CO_2 Emissions
(Time-varying Student's t Markov Copula Estimation)



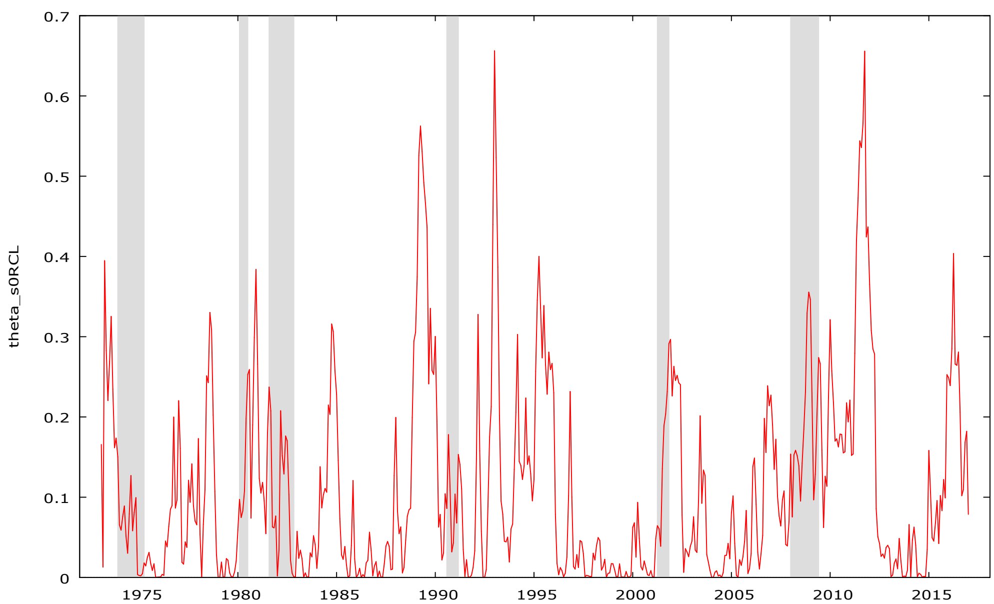
Appendix Figure-II.
Tail Dependence between Industrial Production and CO_2 Emissions
(Time-varying Student's t Markov Copula Estimation)



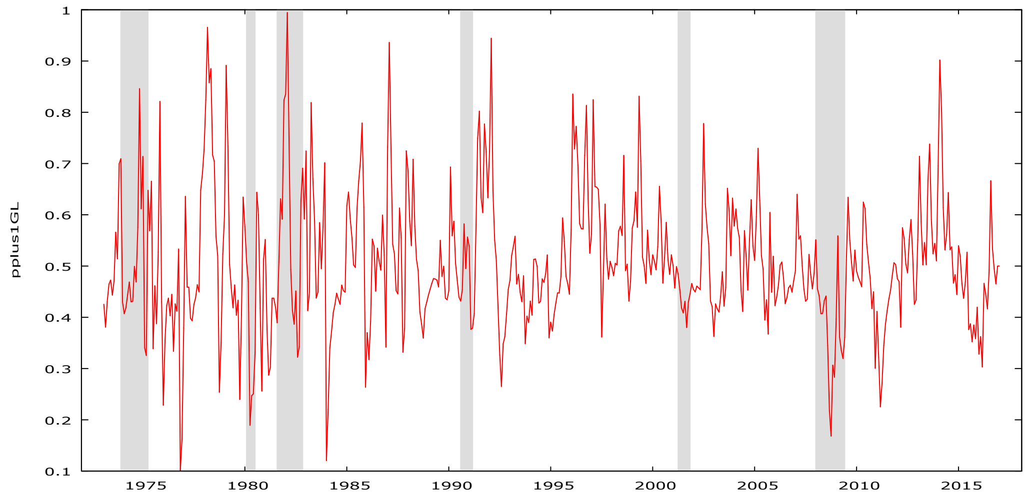
Appendix Figure-III.
Time-varying Dependence between Industrial Production and CO_2 Emissions
(Time-varying Rotated Gumbel Markov Copula Estimation)



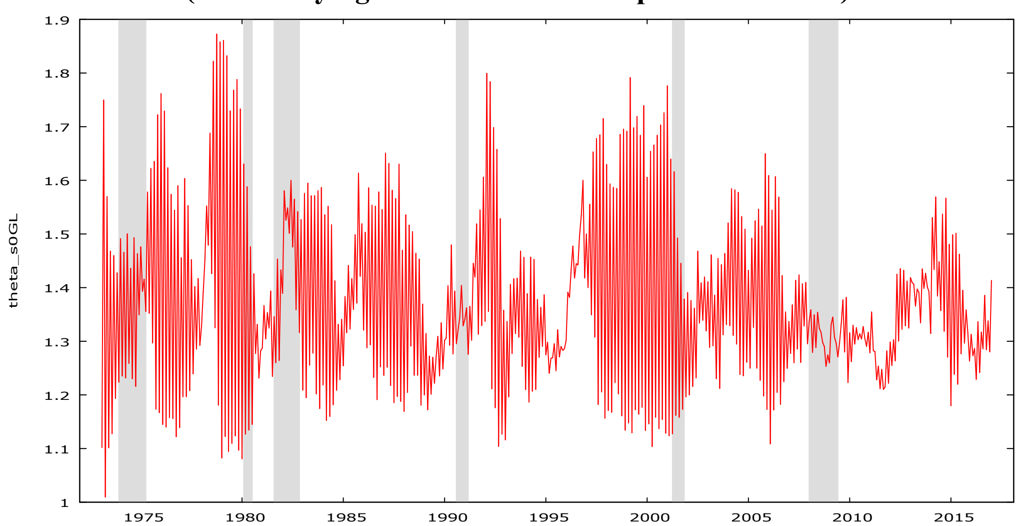
Appendix Figure-IV.
Tail Dependence between Industrial Production and CO_2 Emissions
(Time-varying Rotated Gumbel Markov Copula Estimation)



Appendix Figure-V.
Time-varying Dependence between Industrial Production and CO_2 Emissions
(Time-varying Gumbel Markov Copula Estimation)



Appendix Figure-VI.
Tail Dependence between Industrial Production and CO_2 Emissions
(Time-varying Gumbel Markov Copula Estimation)



Appendix Table-I.
Time-varying Copula Estimation Results for the Industrial Production and CO_2 Emissions

Time-varying Student's t Markov Copula

Parameters	CO_2 Emissions
ω^0	–0.7815 (0.8403)
ω^1	–1.5519 (0.6578)
β	0.4816 (0.1097)
α	2.5191 (1.8223)
P	0.4620 (1.6651)
Q	0.5380 (1.1135)
Log Likelihood	–18.316

Time-varying Rotated Gumbel Markov Copula

Parameters	CO_2 Emissions
ω^0	–0.7815 (0.8403)
ω^1	–1.5519 (0.6578)
β	0.4816 (0.1097)
α	2.5191 (1.8223)
P	0.4620 (1.6651)
Q	0.5380 (1.1135)
Log Likelihood	–17.2728

Time-varying Gumbel Markov Copula

Parameters	CO_2 Emissions
ω^0	1.3105
	(0.1965)
ω^1	0.3710
	(1.2012)
β	–0.8286
	(1.0343)
α	1.3916
	(0.7153)
P	0.7314
	(4.3468)
Q	0.2686
	(8.1723)
Log Likelihood	–20.5714